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New Time Distributions of D^0-$\bar{D}^0$ or B^0-$\bar{B}^0$ Mixing and CP Violation

Zhi-zhong Xing [1]
Sektion Physik, Universität München, Theresienstrasse 37A, 80333 München, Germany

Abstract

The formulae for D^0-$\bar{D}^0$ or B^0-$\bar{B}^0$ mixing and CP violation at the τ-charm or B-meson factories are derived, for the case that only the decay-time distribution of one D or B meson is to be measured. In particular, we point out a new possibility to determine the D^0-$\bar{D}^0$ mixing rate in semileptonic D decays at the $\Psi(4.14)$ resonance; and show that both direct and indirect CP asymmetries can be measured at the $\Upsilon(4S)$ resonance without ordering the decay times of two B_d mesons or measuring their difference.

[1] E-mail: Xing@hep.physik.uni-muenchen.de

1 It is well known that mixing between a neutral meson and its CP-conjugate counterpart can arise if both of them couple to a subset of real and (or) virtual intermediate states. Such mixing effects provide a mechanism whereby interference between the decay amplitudes of two mesons may occur, leading to the phenomenon of CP violation. To date the K^0-$\bar{K}^0$ and B^0_d-$\bar{B}^0_d$ mixing rates have been measured [1], and the CP-violating signals in neutral K-meson decays have unambiguously been established [2]. A preliminary but encouraging result for the observation of CP violation in B^0_d vs $\bar{B}^0_d \rightarrow J/\psi K_{\rm S}$ decay modes has recently been reported by the CDF Collaboration [3]. In contrast, the present experiments have only yielded the upper bound on D^0-$\bar{D}^0$ mixing and the lower bound on B^0_s-$\bar{B}^0_s$ mixing [1], which are respectively expected to be rather small and large in the standard model. Today the B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ systems are playing important roles in the study of flavor mixing and CP violation beyond the neutral kaon system. The D^0-$\bar{D}^0$ system is, on the other hand, of particular interest to probe possible new physics that might give rise to observable D^0-$\bar{D}^0$ mixing and CP violation in the charm sector.

The most promising place to produce B^0_d and $\bar{B}^0_d$ events with high statistics and low backgrounds is the $\Upsilon(4S)$ resonance, on which the asymmetric B-meson factories at KEK and SLAC as well as the symmetric B-meson factory at Cornell are based. Similarly B^0_s and $\bar{B}^0_s$ events may coherently be produced at the $\Upsilon(5S)$ resonance. At a τ-charm factory D^0 and $\bar{D}^0$ events will in huge amounts be produced at the $\Psi(4.14)$ resonance. To measure CP violation on any resonance, where the produced meson pair has the odd charge-conjugation parity ($C = -1$), a determination of the time interval between two meson decays is generally needed. This has led to the idea of asymmetric e^+e^- collisions at the $\Upsilon(4S)$ resonance, i.e., asymmetric B-meson factories, in which the large boost allows to order the decay times of two B_d mesons and to measure their difference.

Recently a new idea, that CP violation can be measured on the $\Upsilon(4S)$ resonance without ordering the decay times of two B_d mesons or determining their difference, has been pointed out by Foland [4]. If this idea is really feasible, it implies that the time-dependent measurement of B^0_d-$\bar{B}^0_d$ mixing and CP violation may be realized at a symmetric e^+e^- collider running at the $\Upsilon(4S)$ resonance, such as the one operated by the CLEO Collaboration at Cornell. It also implies that the time-dependent measurement of D^0-$\bar{D}^0$ mixing and CP violation may straightforwardly be carried out at the $\Psi(4.14)$ resonance with no need to build an asymmetric τ-charm factory. Therefore a further and more extensive exploration of Foland's idea and its consequences is desirable.

This note aims at reformulating the phenomenology of meson-antimeson mixing and CP violation at the $\Upsilon(4S)$, $\Upsilon(5S)$ or $\Psi(4.14)$ resonance, for the case that only the decay-time distribution of one meson is to be measured. We take both $C = -1$ and $C = +1$ cases of the produced meson pair into account, and make no special assumption in deriving the generic formulae. In particular, we point out a new possibility to determine the D^0-$\bar{D}^0$ mixing rate in

semileptonic D decays at the $\Psi(4.14)$ resonance; and show that both direct and indirect CP asymmetries can be measured at the $\Upsilon(4S)$ resonance without ordering the decay times of two B_d mesons or measuring their difference.

2 Let us make use of P to symbolically denote D, B_d or B_s meson. In the assumption of CPT invariance, the mass eigenstates of P^0 and $\bar{P}^0$ mesons can be written as

$$
\begin{aligned}
|P_{\rm L}\rangle &= p|P^0\rangle \ + \ q|\bar{P}^0\rangle \ , \\
|P_{\rm H}\rangle &= p|P^0\rangle \ - \ q|\bar{P}^0\rangle \ ,
\end{aligned} \tag{1}
$$

in which the subscripts "L" and "H" stand for Light and Heavy respectively, and (p, q) are complex mixing parameters. The proper-time evolution of an initially $(t = 0)$ pure P^0 or $\bar{P}^0$ meson is given as

$$
\begin{aligned}
|P^0(t)\rangle &= g_+(t)|P^0\rangle \ + \ \frac{q}{p}g_-(t)|\bar{P}^0\rangle \ , \\
|\bar{P}^0(t)\rangle &= g_+(t)|\bar{P}^0\rangle \ + \ \frac{p}{q}g_-(t)|P^0\rangle \ ,
\end{aligned} \tag{2}
$$

where

$$
\begin{aligned}
g_+(t) &= \exp\left[-\left({\rm i}m + \frac{\Gamma}{2}\right)t\right]\cosh\left[\left({\rm i}\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right] \ , \\
g_-(t) &= \exp\left[-\left({\rm i}m + \frac{\Gamma}{2}\right)t\right]\sinh\left[\left({\rm i}\Delta m - \frac{\Delta\Gamma}{2}\right)\frac{t}{2}\right] \ ,
\end{aligned} \tag{3}
$$

with the definitions $m = (m_{\rm L} + m_{\rm H})/2$, $\Delta m = m_{\rm H} - m_{\rm L}$, $\Gamma = (\Gamma_{\rm L} + \Gamma_{\rm H})/2$, and $\Delta\Gamma = \Gamma_{\rm L} - \Gamma_{\rm H}$. Here $m_{\rm L(H)}$ and $\Gamma_{\rm L(H)}$ are the mass and width of $P_{\rm L(H)}$, respectively. In practice it is more popular to use two dimensionless parameters for the description of P^0-$\bar{P}^0$ mixing: $x = \Delta m/\Gamma$ and $y = \Delta\Gamma/(2\Gamma)$.

For a coherent $P^0\bar{P}^0$ pair at rest, its time-dependent wave function can be written as

$$
\frac{1}{\sqrt{2}}\left[|P^0(\mathbf{K},t)\rangle \otimes |\bar{P}^0(-\mathbf{K},t)\rangle \ + \ C|P^0(-\mathbf{K},t)\rangle \otimes |\bar{P}^0(\mathbf{K},t)\rangle\right] \ , \tag{4}
$$

where $\mathbf{K}$ is the three-momentum vector of the P mesons, and $C = \pm 1$ denotes the charge-conjugation parity of this coherent system. The formulae for the time evolution of P^0 and $\bar{P}^0$ mesons have been given in Eq. (2). Here we consider the case that one of the two P mesons (with momentum $\mathbf{K}$) decays to a final state f_1 at proper time t_1 and the other (with $-\mathbf{K}$) to f_2 at t_2. f_1 and f_2 may be either hadronic or semileptonic states. The amplitude of such a joint decay mode is given by

$$
\begin{aligned}
A(f_1,t_1;f_2,t_2)_C &= \frac{1}{\sqrt{2}}A_{f_1}A_{f_2}\xi_C\left[g_+(t_1)g_-(t_2) + Cg_-(t_1)g_+(t_2)\right] + \\
&\quad \frac{1}{\sqrt{2}}A_{f_1}A_{f_2}\zeta_C\left[g_+(t_1)g_+(t_2) + Cg_-(t_1)g_-(t_2)\right] \ ,
\end{aligned} \tag{5}
$$

where $A_{f_i} = \langle f_i|\mathcal{H}|P^0\rangle$, $\lambda_i = (q/p)(\langle f_i|\mathcal{H}|\bar{P}^0\rangle/\langle f_i|\mathcal{H}|P^0\rangle)$ (for $i = 1, 2$), and

$$\begin{aligned} \xi_C &= \frac{p}{q}\left(1 \; + \; C\lambda_{f_1}\lambda_{f_2}\right) , \\ \zeta_C &= \frac{p}{q}\left(\lambda_{f_2} \; + \; C\lambda_{f_1}\right) . \end{aligned} \tag{6}$$

After a lengthy calculation [5, 6], we obtain the time-dependent decay rate as follows:

$$\begin{aligned} R(f_1, t_1; f_2, t_2)_C \;\; \propto \;\; & |A_{f_1}|^2|A_{f_2}|^2 \exp(-\Gamma t_+) \;\; \times \\ & \Big[\left(|\xi_C|^2 + |\zeta_C|^2\right) \cosh(y\Gamma t_C) - 2\mathrm{Re}\,(\xi_C^*\zeta_C) \sinh(y\Gamma t_C) \\ & - \left(|\xi_C|^2 - |\zeta_C|^2\right) \cos(x\Gamma t_C) + 2\mathrm{Im}\,(\xi_C^*\zeta_C) \sin(x\Gamma t_C)\Big] \; , \end{aligned} \tag{7}$$

where $t_C = t_2 + Ct_1$ has been defined.

Now we integrate the decay rate $R(f_1, t_1; f_2, t_2)$ over $t_1 \in [0, \infty)$, i.e., only the time distribution of P-meson decays into the final state f_2 is kept [4]. The result, with the notation $t_2 = t$, is given as

$$\begin{aligned} R(f_1, f_2; t)_C \;\; \propto \;\; & |A_{f_1}|^2|A_{f_2}|^2 \exp(-\Gamma t) \;\; \times \\ & \left[\frac{|\xi_C|^2 + |\zeta_C|^2}{\sqrt{1-y^2}} \cosh(y\Gamma t + C\phi_y) - \frac{2\mathrm{Re}\,(\xi_C^*\zeta_C)}{\sqrt{1-y^2}} \sinh(y\Gamma t + C\phi_y)\right. \\ & \left. -\frac{|\xi_C|^2 - |\zeta_C|^2}{\sqrt{1+x^2}} \cos\left(x\Gamma t + C\phi_x\right) + \frac{2\mathrm{Im}\,(\xi_C^*\zeta_C)}{\sqrt{1+x^2}} \sin(x\Gamma t + C\phi_x)\right] \; , \end{aligned} \tag{8}$$

where the phase shifts ϕ_x and ϕ_y are defined by $\tan\phi_x = x$ and $\tanh\phi_y = y$, respectively.

The joint decay rate obtained above is a new result and serves as the master formula of this paper. In the following we shall specifically investigate meson-antimeson mixing and CP violation in D- and B-meson decays into the semileptonic final states, the hadronic CP eigenstates, and the hadronic non-CP eigenstates.

3 Let us first consider the joint decays of $(P^0\bar{P}^0)_C$ pairs into two semileptonic states $(l^\pm X_a^\mp)$ and $(l^\pm X_b^\mp)$, i.e., the dilepton events in the final states. Keeping the validity of the $\Delta Q = \Delta P$ rule and CPT invariance, we have $|\langle l^- X_i^+|\mathcal{H}|P^0\rangle| = |\langle l^+ X_i^-|\mathcal{H}|\bar{P}^0\rangle| = 0$ and $|\langle l^+ X_i^-|\mathcal{H}|P^0\rangle| = |\langle l^- X_i^+|\mathcal{H}|\bar{P}^0\rangle| \neq 0$. The latter is denoted later by $|A_{li}|$ for $i = a$ or b. With the help of Eq. (8), we arrive at the same-sign and opposite-sign dilepton rates as follows:

$$\begin{aligned} N_C^{++}(t) \;\; &\propto \;\; \left|\frac{p}{q}\right|^2 |A_{la}|^2|A_{lb}|^2 \exp(-\Gamma t) \left[\frac{\cosh(y\Gamma t + C\phi_y)}{\sqrt{1-y^2}} \; - \; \frac{\cos(x\Gamma t + C\phi_x)}{\sqrt{1+x^2}}\right] , \\ N_C^{--}(t) \;\; &\propto \;\; \left|\frac{q}{p}\right|^2 |A_{la}|^2|A_{lb}|^2 \exp(-\Gamma t) \left[\frac{\cosh(y\Gamma t + C\phi_y)}{\sqrt{1-y^2}} \; - \; \frac{\cos(x\Gamma t + C\phi_x)}{\sqrt{1+x^2}}\right] ; \end{aligned} \tag{9}$$

and

$$N_C^{+-}(t) \;\propto\; 2|A_{la}|^2|A_{lb}|^2 \exp(-\Gamma t) \left[\frac{\cosh(y\Gamma t + C\phi_y)}{\sqrt{1-y^2}} \; + \; \frac{\cos(x\Gamma t + C\phi_x)}{\sqrt{1+x^2}}\right] . \tag{10}$$

Obviously the relationship $N^{++}_{+1}(t) N^{--}_{-1}(t) = N^{++}_{-1}(t) N^{--}_{+1}(t)$ holds.

The measure of CP violation in P^0-$\bar{P}^0$ mixing turns out to be

$$\mathcal{A}^{+-}_C(t) \;=\; \frac{N^{++}_C(t) - N^{--}_C(t)}{N^{++}_C(t) + N^{--}_C(t)} \;=\; \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4} \;, \tag{11}$$

independent of both the decay time t and the charge-conjugation parity C. Within the standard model the magnitude of $\mathcal{A}^{+-}_C(t)$ is estimated to be of $\mathcal{O}(10^{-3})$ or smaller, for either the D^0-$\bar{D}^0$ system [6] or the B^0-$\bar{B}^0$ system [7, 8]. But it might significantly be enhanced if there were new physics contributions to P^0-$\bar{P}^0$ mixing [6 – 9].

On the other hand, the rate of P^0-$\bar{P}^0$ mixing can be determined from

$$\begin{aligned} S^{+-}_C(t) \;&=\; \frac{N^{++}_C(t) + N^{--}_C(t)}{N^{+-}_C(t)} \\ &=\; \frac{1}{2}\left(\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2 \right) \frac{\cosh(y\Gamma t + C\phi_y) - z\cos(x\Gamma t + C\phi_x)}{\cosh(y\Gamma t + C\phi_y) + z\cos(x\Gamma t + C\phi_x)} \;, \end{aligned} \tag{12}$$

where $z = \sqrt{(1-y^2)/(1+x^2)}$. As for $S^{+-}_C(t)$, the approximation $(|p/q|^2 + |q/p|^2)/2 \approx 1$ is rather safe in the standard model.

For the B^0_d-$\bar{B}^0_d$ system we show the dependence of $S^{+-}_C(t)$ on the decay time t in Fig. 1, where $x \approx 0.723$ and $y \approx 0$ [1] (accordingly, $\phi_x \approx 0.626$ and $\phi_y \approx 0$) have been taken. We find that $S^{+-}_{-1}(t)$ and $S^{+-}_{+1}(t)$ become maximal at the positions $\Gamma t = (\pi + \phi_x)/x \approx 5.2$ and $\Gamma t = (\pi - \phi_x)/x \approx 3.5$, respectively. The phase interval between these two line shapes, amounting to $2\phi_x/x$, also measures the rate of B^0_d-$\bar{B}^0_d$ mixing [2].

For the D^0-$\bar{D}^0$ system one has the following conservative bound on the mixing rate: $x < 0.1$ and $y < 0.1$ (satisfying $x^2 + y^2 < 0.01$), which were obtained from the wrong-sign semileptonic decays of neutral D mesons at the 90% confidence level [1, 11]. The relative magnitude of x and y remains unclear, as the theoretical estimates involve too large uncertainty due to the long-distance effects [12]. In Fig. 2 we illustrate the time-dependent behavior of $S^{+-}_C(t)$ with three types of inputs: (a) $x \approx y \approx 0.06$; (b) $x \approx 0.08$ and $y \approx 0$; and (c) $x \approx 0$ and $y \approx 0.08$. We see that the line shape of $S^{+-}_C(t)$ for the $x \ll y$ case is clearly distinguishable, when $\Gamma t \geq 5$, from that for the $x \gg y$ case. A delicate analysis even allows to discern the relative magnitude of x and y. This provides us a new possibility, different from those proposed previously in the literature [13], to measure the rate of D^0-$\bar{D}^0$ mixing [3].

[2] The $C = +1$ $B^0_d \bar{B}^0_d$ events can in practice be produced just above the $\Upsilon(4S)$ energy threshold, i.e., above $M_B + M_{B^*}$ but below $2M_{B^*}$, whereby the B^{*0}_d and $\bar{B}^{*0}_d$ mesons decay radiatively, leaving $B^0_d \bar{B}^0_d \gamma$ with the $B^0_d \bar{B}^0_d$ pair in the $C = +1$ state. In this case one has to pay for the cost that the $b\bar{b}$ cross section above the $\Upsilon(4S)$ resonance is smaller than that on the resonance [10].

[3] For τ-charm factories running at the $\Psi(4.14)$ resonance, the coherent $D^0\bar{D}^0$ events can be produced through the transitions $\Psi(4.14) \rightarrow \gamma (D^0\bar{D}^0)_{C=+1}$ and $\Psi(4.14) \rightarrow \pi^0 (D^0\bar{D}^0)_{C=-1}$. Note that the $C = -1$ $D^0\bar{D}^0$ events can also be produced from the decay of the $\Psi(3.77)$ resonance [14].

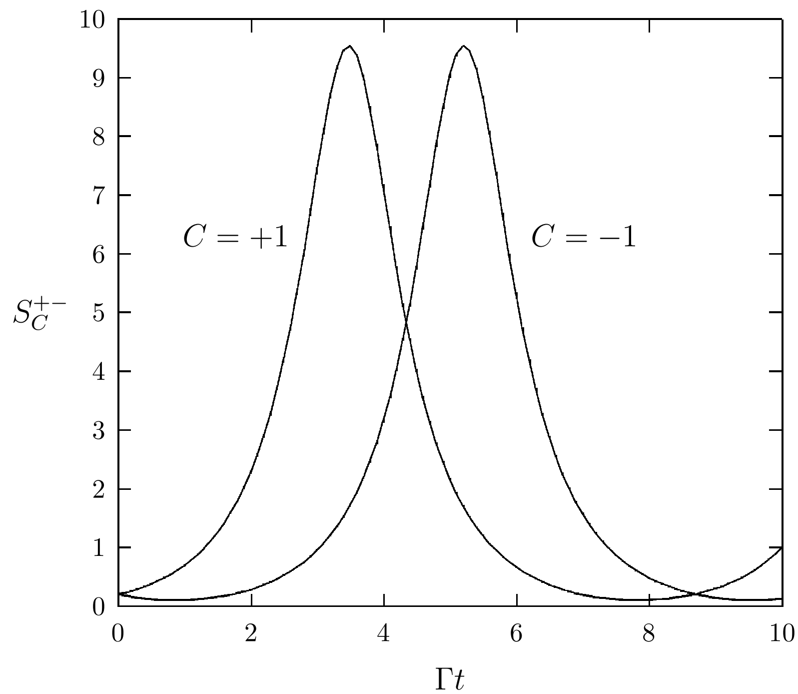

Figure 1: Ratios of the same-sign to opposite-sign dilepton events changing with the decay time t at the $\Upsilon(4S)$ resonance, where $x \approx 0.723$ and $y \approx 0$ have been taken.

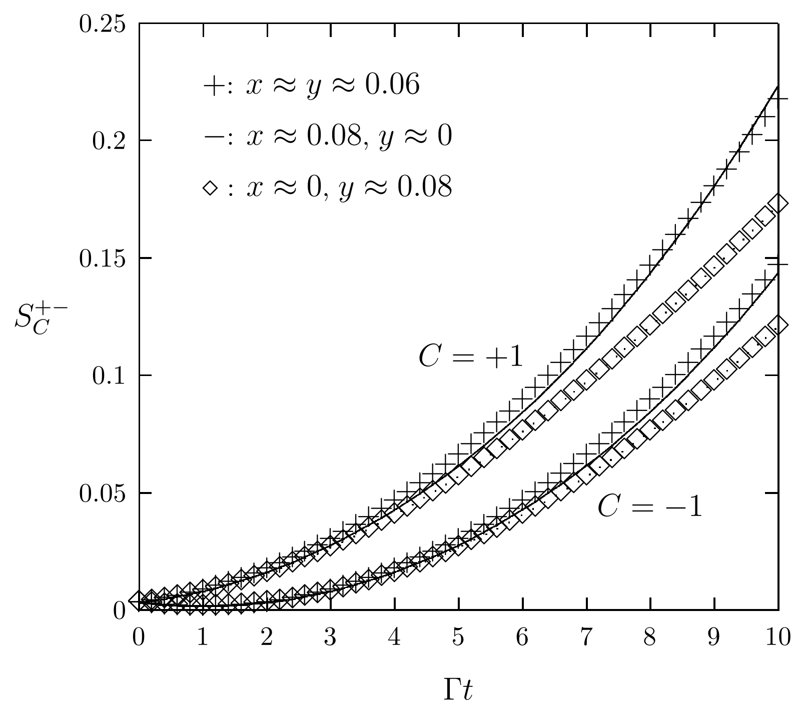

Figure 2: Illustrative plot for ratios of the same-sign to opposite-sign dilepton events changing with the decay time at the $\Psi(4.14)$ resonance.

For the B_s^0-$\bar{B}_s^0$ system we have $x > 14$ from current experimental data at the 95% confidence level [1], and $y \sim 0.03$ from the latest theoretical calculation [15]. Hence the behavior of $S_C^{+-}(t)$ depends mainly upon the value of x. Taking $x \approx 20$ and $y \approx 0$ typically, one finds that the oscillation term of $S_C^{+-}(t)$ is suppressed by a factor $z \approx 1/x$. As a consequence $S_C^{+-}(t) \approx 1$ holds for variable values of x, i.e., the magnitude of $S_C^{+-}(t)$ deviates little from unity. This property makes it somehow difficult to determine the precise value of x by measuring the time distribution of $S_C^{+-}(t)$ at the $\Upsilon(5S)$ resonance [16].

4 Now let us consider CP violation in neutral B- or D-meson decays into hadronic CP eigenstates at the $\Upsilon(4S)$ or $\Psi(4.14)$ resonance. In this case the semileptonic decay of one P meson serves to tag the flavor of the other P meson decaying into a nonleptonic CP eigenstate. There are generally three different types of CP asymmetries, arising from P^0-$\bar{P}^0$ mixing itself, from the interference between two decay amplitudes (*direct* CP violation), and from the interplay of decay and P^0-$\bar{P}^0$ mixing (*indirect* CP violation). For the B_d^0-$\bar{B}_d^0$ system the typical magnitudes of these three kinds of CP-violating effects are respectively expected to be of $\mathcal{O}(10^{-3})$, $\mathcal{O}(10^{-2})$ to $\mathcal{O}(10^{-1})$, and $\mathcal{O}(1)$ in the standard model. It is more difficult to classify the magnitudes of direct and indirect CP asymmetries in different decay channels of neutral D or B_s mesons, but CP violation in either B_s^0-$\bar{B}_s^0$ or D^0-$\bar{D}^0$ mixing is anticipated to be below $\mathcal{O}(10^{-3})$ within the standard model. Therefore the neglect of tiny mixing-induced CP violation, equivalent to taking $|q/p| \approx 1$ (as well as $y \approx 0$), is a good approximation when we calculate the direct and indirect CP asymmetries in most B_d, B_s and D decays. We obtain the time-dependent decay rates as

$$\begin{aligned} R(l^\pm, f; t)_C \;\; \propto \;\; & |A_l|^2 |A_f|^2 \exp(-\Gamma t) \left[\left(1 + |\lambda_f|^2\right) \pm \frac{1 - |\lambda_f|^2}{\sqrt{1+x^2}} \cos(x\Gamma t + C\phi_x) \right. \\ & \left. \mp \frac{2\mathrm{Im}\lambda_f}{\sqrt{1+x^2}} \sin(x\Gamma t + C\phi_x) \right] , \end{aligned} \tag{13}$$

where f is the CP eigenstate, and $\lambda_f = (q/p)\langle f|\mathcal{H}|\bar{P}^0\rangle / \langle f|\mathcal{H}|P^0\rangle$ as defined before. The CP asymmetry is then given by

$$\begin{aligned} \mathcal{A}_f^C(t) & = \frac{R(l^-, f; t) - R(l^+, f; t)}{R(l^-, f; t) + R(l^+, f; t)} \\ & = \frac{1}{\sqrt{1+x^2}} \left[\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(x\Gamma t + C\phi_x) - \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2} \sin(x\Gamma t + C\phi_x) \right] . \end{aligned} \tag{14}$$

Clearly $\mathcal{A}_f^C(t)$ consists of both the direct CP asymmetry ($|\lambda_f| \neq 1$) and the indirect one ($\mathrm{Im}\lambda_f \neq 0$). Measuring the time distribution of $\mathcal{A}_f^C(t)$ can distinguish between these two sources of CP violation.

For illustration let us take the gold-plated channels B_d^0 vs $\bar{B}_d^0 \rightarrow J/\psi K_S$, which are dominated by the tree-level quark transitions [17], for example. It is well known that $|\lambda_{\psi K_S}| \approx 1$ and $\mathrm{Im}\lambda_{\psi K_S} = \sin(2\beta)$ hold, where $\beta = \arg[-(V_{cb}^* V_{cd})/(V_{tb}^* V_{td})]$ is an inner angle of the quark

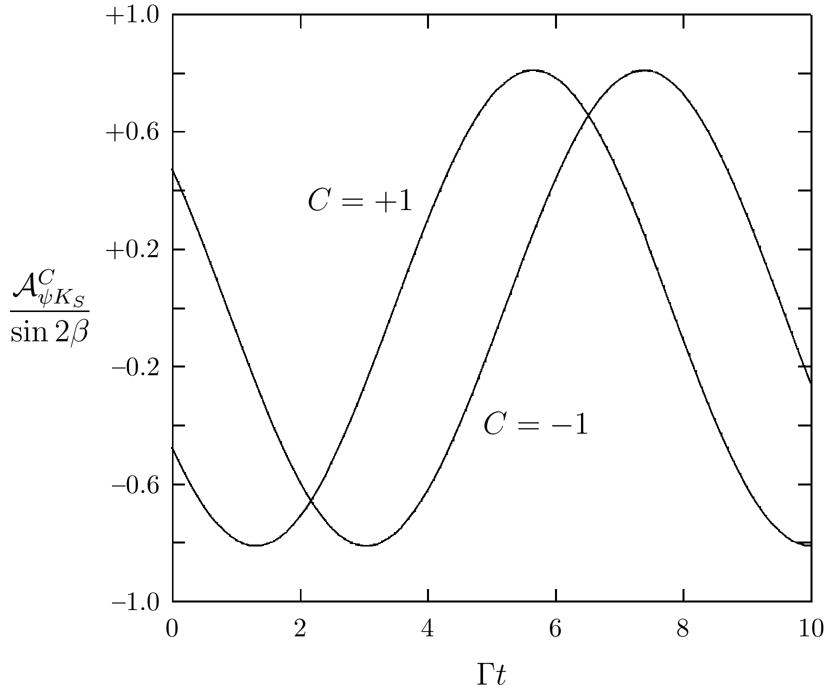

Figure 3: Time-dependent behavior of the CP asymmetry in B^0_d vs $\bar{B}^0_d \rightarrow J/\psi K_S$ decays, where $x \approx 0.723$ has been taken.

mixing unitarity triangle. We are left with

$$\mathcal{A}^C_{\psi K_S}(t) \;=\; -\frac{\sin 2\beta}{\sqrt{1+x^2}} \sin(x\Gamma t + C\phi_x) \;, \tag{15}$$

to a high degree of accuracy. The behavior of this CP asymmetry changing with the decay time t is illustrated in Fig. 3. Certainly the weak phase β can well be determined from such a time-dependent measurement at the $\Upsilon(4S)$ resonance [4].

5 Finally we consider the case that both P^0 and $\bar{P}^0$ mesons decay into a common non-CP eigenstates. For neutral D-meson decays, most of such decay modes occur through the quark transitions $c \rightarrow s(u\bar{d})$ and $c \rightarrow d(u\bar{s})$ or their flavor-conjugate processes. For B_d and B_s decays, most of such decay channels take place through the quark transitions $b \rightarrow q(u\bar{c})$ and $b \rightarrow q(c\bar{u})$ or their flavor-conjugate processes (for $q = d$ or s). The typical examples of such decay channels include D^0 vs $\bar{D}^0 \rightarrow K^{\pm}\pi^{\mp}$, B^0_d vs $\bar{B}^0_d \rightarrow D^{\pm}\pi^{\mp}$, and B^0_s vs $\bar{B}^0_s \rightarrow D^{\pm}_s K^{\mp}$ decays [5].

For simplicity we concentrate only on the decay modes in which no direct CP violation exists, i.e., the decay amplitudes of $P^0 \rightarrow f$ and $\bar{P}^0 \rightarrow \bar{f}$ are governed by a single weak phase. We also take $y \approx 0$, as indirect CP violation is primarily associated with the mixing parameter x. For coherent $P^0\bar{P}^0$ decays at the resonance, we make use of the semileptonic decay of one P

[4]The result for the $C=-1$ case has been presented in Ref. [4], where the definition of CP asymmetries is different from ours in Eq. (14).

[5]To extract the weak phase β and β' a study of B_d and B_s decays into the non-CP eigenstates $D^{*\pm}D^{\mp}$ and $D^{*\pm}_s D^{\mp}_s$, in which the penguin effects are negligibly small, is also of particular interest [18].

meson to tag the flavor of the other P meson decaying into f or $\bar{f}$. The time-dependent rates of such joint decay modes, with the help of Eq. (8), are given as follows:

$$
\begin{aligned}
R(l^-, f; t)_C &\propto |A_l|^2 |A_f|^2 \exp(-\Gamma t) \left[\left(1 + |\lambda_f|^2\right) + \frac{1 - |\lambda_f|^2}{\sqrt{1+x^2}} \cos(x\Gamma t + C\phi_x) \right. \\
& \left. \qquad - \frac{2 \mathrm{Im} \lambda_f}{\sqrt{1+x^2}} \sin(x\Gamma t + C\phi_x) \right] , \\
R(l^+, \bar{f}; t)_C &\propto |A_l|^2 |A_f|^2 \exp(-\Gamma t) \left[\left(1 + |\bar{\lambda}_{\bar{f}}|^2\right) + \frac{1 - |\bar{\lambda}_{\bar{f}}|^2}{\sqrt{1+x^2}} \cos(x\Gamma t + C\phi_x) \right. \\
& \left. \qquad - \frac{2 \mathrm{Im} \bar{\lambda}_{\bar{f}}}{\sqrt{1+x^2}} \sin(x\Gamma t + C\phi_x) \right] , \qquad (16)
\end{aligned}
$$

where $\bar{\lambda}_{\bar{f}} = (p/q) \langle \bar{f} | \mathcal{H} | P^0 \rangle / \langle \bar{f} | \mathcal{H} | \bar{P}^0 \rangle$, and the relationship $|\bar{\lambda}_{\bar{f}}| = |\lambda_f|$ holds. The time-dependent CP asymmetry turns out to be

$$
\begin{aligned}
\mathcal{A}^C_{f\bar{f}}(t) &= \frac{R(l^-, f; t) - R(l^+, \bar{f}; t)}{R(l^-, f; t) + R(l^+, \bar{f}; t)} \\
&= \frac{\mathrm{Im} \left(\bar{\lambda}_{\bar{f}} - \lambda_f\right) \sin(x\Gamma t + C\phi_x)}{\sqrt{1+x^2}\,(1 + |\lambda_f|^2) + F(\lambda_f, \bar{\lambda}_{\bar{f}}, x\Gamma t + C\phi_x)} , \qquad (17)
\end{aligned}
$$

in which F is a function defined by $F(z_1, z_2, z_3) = (1 - |z_1|^2) \cos z_3 - \mathrm{Im}(z_1 + z_2) \sin z_3$. Note that only the difference between $\mathrm{Im}\bar{\lambda}_{\bar{f}}$ and $\mathrm{Im}\lambda_f$, which would vanish if the relevant weak phase were zero, measures the CP violation.

Taking the decay modes B^0_d vs $\bar{B}^0_d \rightarrow D^\pm \pi^\mp$ for example, one finds that measuring the CP violating quantity $\mathrm{Im}(\bar{\lambda}_{D^\pm\pi^\mp} - \lambda_{D^\mp\pi^\pm})$ allows the determination of the weak phase $(2\beta + \gamma)$, where $\gamma = \arg[-(V^*_{ub}V_{ud})/(V^*_{cb}V_{cd})]$ is another angle of the quark mixing unitarity triangle [19]. This illustates that some attention is worth being paid to CP violation in neutral B- and D-meson decays into hadronic non-CP eigenstates.

6 In summary, we have derived the generic formulae for P^0-$\bar{P}^0$ mixing and CP violation at the resonance where $P^0\bar{P}^0$ pairs can coherently be produced, for the case that only the decay-time distribution of one P meson is to be measured. Examples for the D^0-$\bar{D}^0$, B^0_d-$\bar{B}^0_d$ and B^0_s-$\bar{B}^0_s$ systems are discussed. In particular, we point out a new possibility to measure D^0-$\bar{D}^0$ mixing in semileptonic D-meson decays at the $\Psi(4.14)$ resonance, and show that both direct and indirect CP asymmetries can be determined at the $\Upsilon(4S)$ resonance with no need to order the decay times of two B_d mesons or to measure their difference.

We expect that the formulae and examples presented here will be useful for the physics being or to be studied at the B-meson and τ-charm factories.

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